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Particle migrations in suspension flows

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An apparatus is described whereby the distribution of particles in a neutrally buoyant suspension of spheres flowing through a tube may be measured. The discovery of a narrow region of high concentration near to the tube wall, and the behaviour of this 'wall peak' at various concentrations and positions along the length of the tube are described. An explanation for this behaviour is put forward in terms of the formation of the peak by interaction of spheres with the tube entrance, and of the formation within this peak of necklace-like strings of particles.

1. Introduction

Many theoretical and experimental investigations have been made into the behaviour of solid/liquid suspensions when flowing in capillary tubes, and particularly into the particle migrations which are observed or inferred to have occurred.

The results of Segré & Silberberg (1961, 1962), in which the 'tubular pinch effect' was first demonstrated for a macroscopic suspension of rigid spheres, clearly showed the existence of particle migrations perpendicular to the direction of flow within Poiseuille flow. This process in which all particles entering the mouth of the flow tube were observed to congregate at a radial position of $r = 0{\cdot}6R$ (where R is the radius of the tube) was shown to arise as a result of the inertia of the fluid. Oliver (1962), by studying the effects of particle rotation, suggested that this 'tubular pinch effect' may be explained in terms of an outwardly directed 'Magnus effect' away from the tube axis, and an inwardly directed 'wall effect' caused by hydrodynamic interaction between the particles and the tube wall. More recent work which also demonstrates actual particle migrations within tube flow includes that of Goldsmith & Mason (1961), Jeffrey & Pearson (1965) and Eichhorn & Small (1964).

During their experiments Segré & Silberberg (1962) noted the existence of what were termed 'necklace-like formations of particles' which were observed to form within the flow where the 'tubular pinch effect' was set up. The occurrence of these chains was studied by Segré (1963), who found that their frequency of occurrence depended linearly on particle concentration and dimension, but it appears that no theory has yet been given to evaluate their importance as a feature of the flow, or to show to what extent they may be responsible for the tubular pinch.

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In order to explain anomalous results on the viscosity of suspensions when measured in capillary tubes, Maude & Whitmore (1956) considered the inward displacement of particles which must occur at the mouth of the flow tube by mechanical interaction of the particles and the tube wall.

It was in order to study the relative importance of the above features that the present study was proposed. In §2, the apparatus employed will be described, together with a brief description of the techniques involved. §3 will be devoted to a description of what appears to be a phenomenon not previously described which we have termed the 'wall peak', but which it will be shown is to be expected on the basis of the work of Maude & Whitmore (1956). Finally in §4 a study of the particle chains or necklace will be given and a physical model suggested on the basis of which the results of the present study may be explained.

2. Apparatus and experimental methods

The technique employed in the present study has been that of intersecting light beams (cf. Segré & Silberberg 1962), whereby the particle passages through the intersection of two light beams are detected and counted electronically. Such 'single particle' or 'hit' counts are detected in two suitably situated photoconductive devices simultaneously, and are an indication of the local particle concentration at the beam intersection. Simultaneous or coincidence pulses from the photo-devices are also caused by two or more particles simultaneously but quite separately interrupting the beams, and are known as 'chance' coincidences. Any count of simultaneous pulses over a known period is therefore made up of two components, i.e.

$$N_t = N_h + N_c, \tag{1}$$

where N_t is the total number of coincident counts per unit time, N_h is the number of 'single particle' or 'hit counts' per unit time and N_c is the number of 'chance counts' per unit time. Thus in any study it is necessary that either the possibility of chance counts is eliminated or that the number of those which occur is estimated. In the present study an estimation of chance counts was made experimentally in the following manner. If the two beams of light do not in fact intersect but are separated by some small distance in the direction of flow, then it is seen that 'hit' coincidences cannot possibly occur, while the occurrence of 'chance counts' remains effectively the same. Thus by separating the beams a small distance an estimate of N_c is possible. This displacement was performed electronically rather than mechanically by introducing a delay into one or other of the light beam channels.

The light beams employed were effectively of a focused beam nature rather than those of square cross-section employed by Segré & Silberberg (1962), and enabled reliable scanning to be made over rather small areas at various positions on a tube radius.

The apparatus can be conveniently considered as consisting of three main sections, the flow system, the optical scanner and the electronic counters.

The flow system is shown diagrammatically in figure 1. This arrangement involves a method of continuous pumping of the suspension, which is forced through the flow tube from reservoir C_1 to reservoir C_2. After flowing into an open tank at R the suspension is then returned to C_1 by means of the pump P. This pump is a flow inducer made by Watson-Marlow which works on the principle of squeezing an elastic tube between a set of rotating rollers and a semicircular track. While this has the advantage that the suspension at no time comes into contact with any moving parts or valves, it does suffer from the fact that

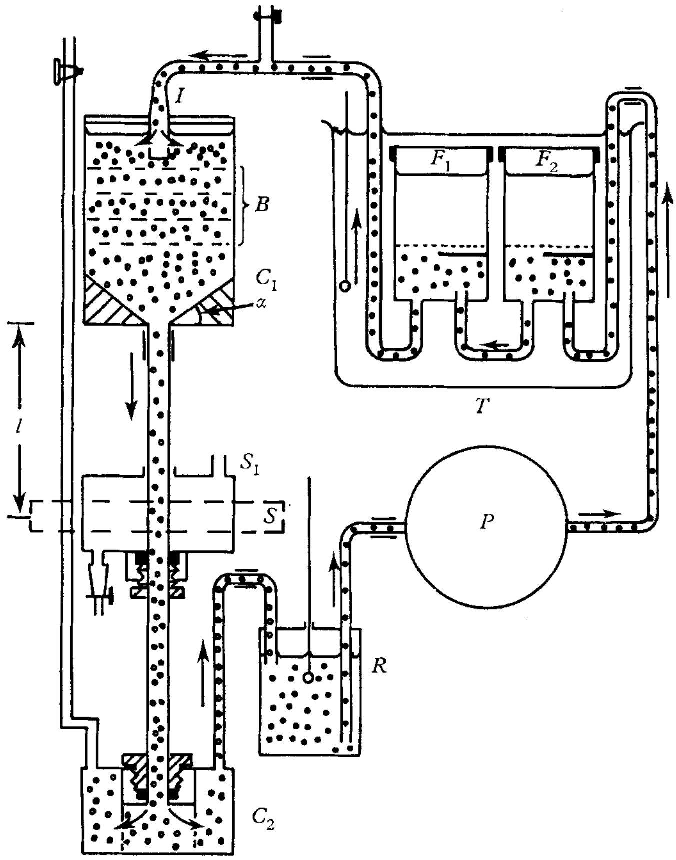

FIGURE 1. Diagram of the apparatus. B, turbulence-removing grid; C_1, inlet reservoir; F_1, F_2, flow-smoothing bottles. S, optical scanner as in figure 3. T, thermostatically controlled tank; P, Watson-Marlow pump; R, open reservoir; S_1, observation section; I, turbulence-producing inlet to C_1.

the flow produced is not smooth but pulsating. This necessitates the use of the smoothing bottles F_1 and F_2 situated at the outlet to the pump, which gave a smoothed flow in all cases considered, such that no jerky motion of the particles could be observed even with the use of high-speed cinematography.

Thorough mixing of the suspension is effected by forcing the suspension through a series of small holes in the inlet tube I thereby resulting in turbulence in the upper part of C_1. Before the suspension enters the flow tube it is necessary that this turbulence be removed so that it first flows through a series of gauzes as shown (B). This ensures that the suspension entering the flow tube from C_1 is as homogeneously mixed as possible yet without apparent turbulence.

The flow tube itself is a glass tube chosen because of its uniformity of internal diameter and also the unscratched nature of its surface. A piece of soda-glass tubing was found to be most suitable, and its internal diameter, found by weighing columns of water drawn off, was found to be $16{\cdot}9 \pm 0{\cdot}10$ mm. As shown in figure 1 the entrance mouthpiece to the tube is a cone of angle 45°, although earlier experiments with other shaped mouthpieces revealed very little difference in the resulting particle distributions.

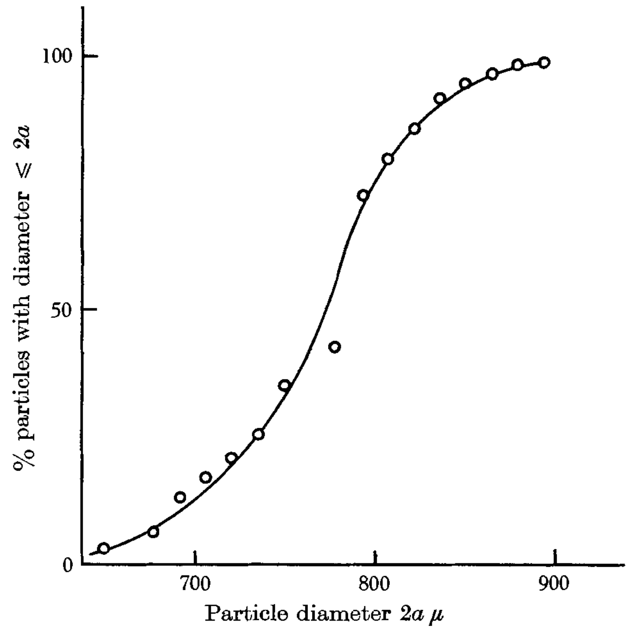

FIGURE 2. Cumulative size distribution of spheres used.

The suspension is made up of spheres of styrene divinyl benzene copolymer in a mixture of water, butane-1,3-diol and glycerol of the same density as the particles. The particles were sieved several times in air and under water before the fraction between 810 and 750 microns was extracted. Calibrated photographs gave the mean particle diameter as 0·779 mm and the mean particle volume as $0{\cdot}246 \times 10^{-3}\,\text{cm}^3$, this latter being measured from the average value of a^3 (a = particle radius). The distribution of sizes is shown in figure 2.

A Ferranti portable viscometer of the rotating cylinder type was used to measure the suspending medium viscosity, while the particle concentration was measured by counting the numbers of particles in known weights of suspension.

In order to keep the suspension viscosity as constant as possible, the temperature of the suspension was kept constant to within 0·1 degC. This was performed by immersing the smoothing bottles F_1 and F_2 in a thermostatically controlled water bath T, and also keeping the room temperature as steady as possible. By these means it was possible to maintain a steady and homogeneous flow of suspension through the system for periods of up to 4 hours, thereby allowing readings of particle distribution to be made.

The optical scanner which was designed such that particle passages might be recorded at any point within the flow tube is shown diagrammatically in figure 3. L_1, L_2 and L_3 are convex lenses of 75 mm focal length, S are light sources, being 6 W filament bulbs powered by a 6 V stabilized supply, G are ground-glass screens to give more even dispersion of the light from S, H are pinholes of selected dimension and P are Mullard OCP 71 phototransistors.

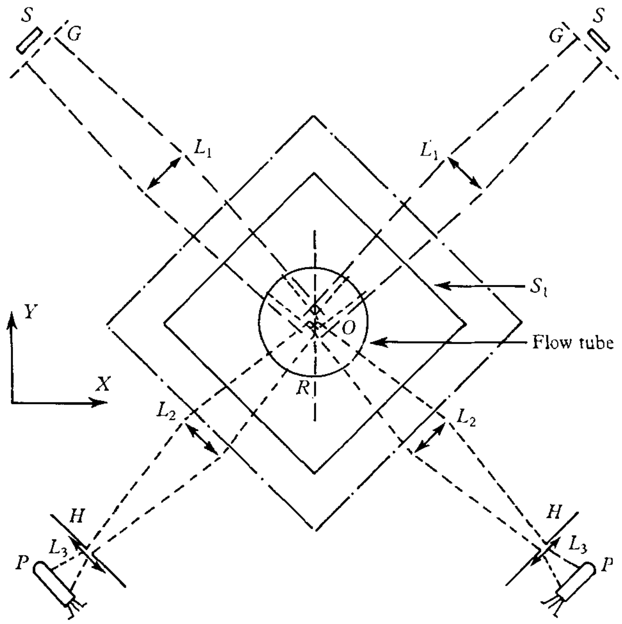

FIGURE 3. Optical scanner. S, lamps; L_1, L_2, L_3, 75 cm focal length lenses; O, point under observation; H, pinholes; P, phototransistors; OR, line of scan; S_1, square tank around region of tube under observation; G, ground-glass screens.

The sections including L_1, G and S are the collimators, being deliberately thrown out of focus to produce an area of general illumination around the point under investigation, in this case at O. This point is defined by the pick-up sections where L_2 and H are adjusted so that an object at O forms an image at H, which means that H produces an image at O. Hence the area over which counts are made is defined by size of the pinhole at H. Lens L_3 is situated immediately behind pinhole H so that the cone of light behind this lens has a cross-section of even illumination. Any light in the beam which is cut off by a particle passage therefore results in a reduction in the proportion of light falling on the phototransistor evenly across the surface of the semi-conductor. This makes the pulse shape practically independent of the initial illumination or the unevenness of the response of the phototransistor across its sensitive area. A square, water-filled, Perspex tank surrounds the flow tube at the region under examination, so that optical distortion due to refraction at the surface of the tube is small.

The proportion of light cut off by a particle passage is determined initially by three factors, the particle centre position with respect to the point under investigation, the particle dimension and the pinhole size. It is seen that in figure 3 a particle centre passing through O will cause the maximum proportion of light to be cut off in both beams and that this will be total if the area defined by the image of the pinhole is equal to or less than the diameter of the particle. Consideration of the various effects of varying the size of the pinhole and the sensitivity of the electronic counters have been fully considered elsewhere (Yearn 1966) and it was decided to use a pinhole which defined an area in the flow tube of the same order of size as the particles. The electronic counting circuits were then arranged so that particles causing a cut-off of about half of the total light would be counted. The area over which hit counts can be recorded will be referred to as the sensitive area and its value must be estimated in any quantitative study, but in the present work it was less than half a sphere diameter wide.

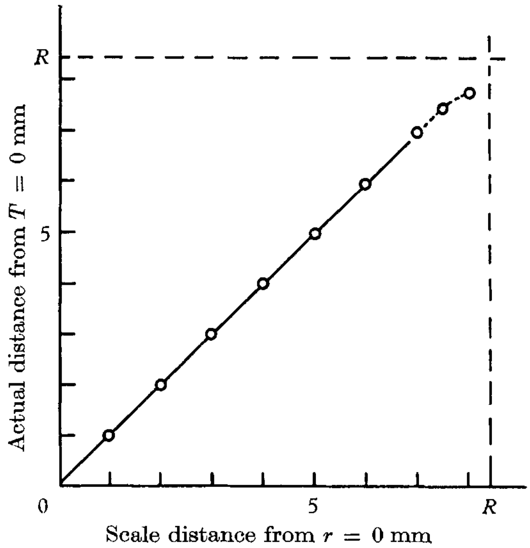

FIGURE 4. Calibration of scan position screw.

The entire optical arrangement was mounted on two sets of tracks at right angles, the movement in the directions x and y being governed by two accurately machined screw threads of 1 mm pitch. It was possible after initial adjustments were complete to move the point of investigation along a radius by use of one only (y), when a scan was made along the radius marked OR. This particular screw thread was provided with a graduated scale such that readings were possible down to $\frac{1}{8}$ mm separations.

The focusing and alignment of the optical system was carried out using a graticule which was lowered into the flow tube. This process was a rather tedious one of trial and error, it being impossible to keep the focus sharp over the whole radius. It was found that a satisfactory compromise was obtained when the focus was sharp at a midpoint, but that when only a small region near to the tube wall was to be examined the adjustment could be made more satisfactorily for this specific region. The graticule was also used to calibrate the scale on the y traversing screw, thus checking that refraction at the wall of the tube did not upset the optical alignment (figure 4).

The main disadvantage in use of this method of scanning lies in the difficulties associated with the elimination of optical distortion. The scanner described has been used in the present study and the results obtained under various conditions which will be presented in later sections suggest that the scanner itself is in no way responsible for the observed phenomena. These validating results include the exhibition of a parabolic velocity profile at points near to the mouth of the tube under certain conditions and the confirmation of certain phenomena by photographic means. The advantages of the method lie in the fact that the flow is in no way hindered by the scanner and that the use of a focused beam technique allows measurement to be made over relatively small areas and at higher concentrations than in the case of Segré & Silberberg (1962).

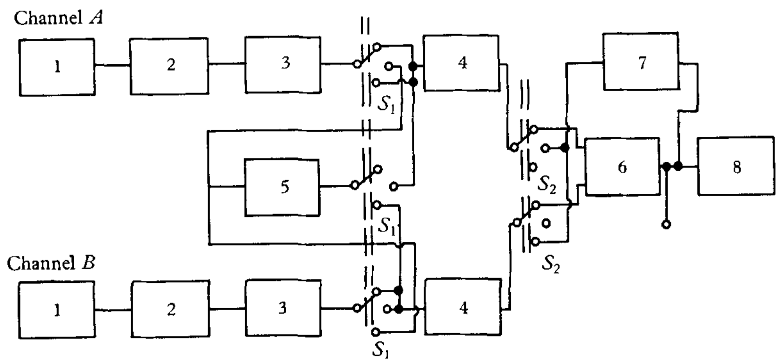

FIGURE 5. Block diagram of electronics. 1, phototransistor; 2, amplifier; 3, Schmidt trigger; 4, monostable multivibrators; 5, delay unit; 6, coincidence counter; 7, phase invertor.

The electronic circuits, a block diagram of which is shown in figure 5, essentially perform the function of counting simultaneous pulses from the phototransistors, although several features have been incorporated in the design which allow determination of 'chance counts' and use over a wide range of conditions.

Pulses from phototransistors (1) are amplified in 3-stage d.c. amplifiers (2) whose outputs are regulated by means of variable resistor networks. These pulses are squared by Schmidt triggers (3) which also act as discriminators in that they will only accept pulses greater than a certain minimum voltage (i.e. the discrimination level). The resulting square pulses are then used to trigger monostable multivibrators (4). Such an arrangement means that particles which cut off more than a certain percentage of light will give a squared output from the monostable multivibrator which is of known height and duration, while those which cut off less than this pre-set percentage will produce no output.

The pulses from these two channels are combined in the simple coincidence unit (6) which gives an output only when two simultaneous input pulses occur. These are connected to a set of scalars (8) which count the pulses. Unit (7) is a simple phase invertor provided so that in conjunction with switch S_2 either channel can be connected directly to the counters bypassing the coincidence circuit.

Switch S_1 allows a variable delay unit (5) to be introduced into either channel in order to measure the numbers of 'chance counts' as previously described. The delay consists essentially of a monostable multivibrator similar to that in unit (4), the pulse length of which determines the delay introduced.

Before this arrangement can be used to measure N_t, N_c and hence N_h it is necessary that three controls be set.

(i) The gain control which effectively determines the 'discrimination level'. This in turn determines the 'sensitive area' over which counts are made, and also the numbers of 'chance counts'. It must therefore be arranged such that when more than a certain percentage of light in each channel is cut off a pulse is emitted by the monostable multivibrators (4). Use of too high a 'discrimination level', while reducing 'chance counts', also reduces the area over which counts are made to such an extent that the number of counts involved is small, so that large statistical errors are introduced. On the other hand, if the 'discrimination level' is too low 'chance counts' become so numerous that they swamp the number of

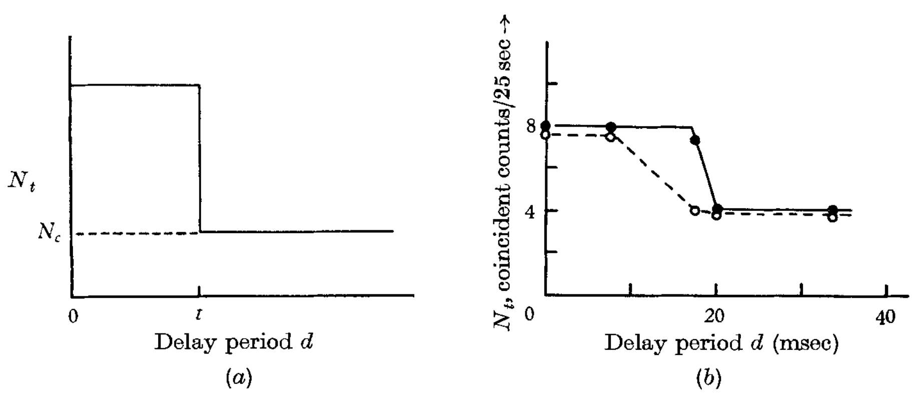

FIGURE 6. (a) Theoretical graph of count against delay. (b) Practical graph of count against delay when the apparatus is not properly aligned.

'hit counts'. It was therefore found convenient to use a discrimination level at 50 % cut-off, which was set by moving an exposed photographic negative across the light beam in each channel and setting the gain control until the scalars just counted.

(ii) The pulse length of the monostable unit 4, because of its finite length, is partially responsible for determining the area over which counts are made. This must therefore be chosen such that it is not so long as to increase the numbers of chance counts unduly, or alternatively too short so that some 'hit counts' are not recorded. It was therefore decided in the present case to employ a value of $t = 10$ msec, which fell roughly half way between the highest and lowest pulse lengths obtained from the phototransistor in the present study.

(iii) The delay period, d. If the numbers of coincident counts are measured when various delays are introduced into one of the channels a graph similar to figure 6 (a) is to be expected where total coincidences (i.e. 'hits' plus 'chance') will still occur up to a delay of period t, the monostable pulse length. In order that 'chance counts' only should be recorded it is therefore necessary that the delay be large enough to bring the counts on to the lower part of the curve. If various delays are introduced into each channel the numbers of counts at each value of

d should coincide as long as the beams are correctly aligned in the direction of flow. If this is not so a graph similar to that of figure 6 (*b*) is obtained where the beam in channel A is observed to be slightly higher than that in channel B. This provides a useful method of checking the lining up at any time during an experimental scan. In the present study a value $d = 33$ msec was employed which always resulted in the measurement of chance counts only.

At each particular point along a radius of the tube it is required to measure N_t the total coincident count and N_c the number of chance counts, occurring within a particular period of time. The value of this period of time is determined by two main factors. First the necessity of using periods long enough to keep the experimental scatter to a minimum, and secondly the desire to keep the time involved in scanning a radius short enough so that long-term drifts do not become apparent. In the case of the first requirement it is preferable to use as long a period as possible but this would severely restrict the number of readings possible in the time available, whereas too short a period would yield too few counts to be statistically reliable. In the experiments reported here readings were taken over a minimum of three such periods of 50 sec. In order that the effects of any long-term drifts be minimized the order in which readings were taken was decided upon by choosing values of r at random rather than moving across a radius in successive steps.

In general readings were taken at $\frac{1}{2}$ mm intervals, although at points of large concentration gradients this was reduced to $\frac{1}{4}$ or even $\frac{1}{8}$ mm separations.

Such a process performed for N_t and N_c allows determination of 'hit counts' N_h at discrete values of r across any desired tube radius, which can then be represented graphically. In order that the particle distribution could be represented continuously, a curve was fitted to these discrete values by calculating the best-fitting 5th-order polynomial in r^2. This was performed by the method of least squares on an IBM 1620 digital computer using a standard Fortran Library program (L 70001), although this was only carried out for the central region of the tube where no sharp concentration gradients occurred; the curve through regions where such gradients existed were drawn in later by eye.

Using the apparatus in the manner described it was possible to measure the distribution of the number of particles (crossing in unit time a given area) along a radius of the tube at various distances from the tube mouth.

3. The wall peak

In order to check operation of the apparatus described in the previous section, it was hoped to demonstrate that the flow pattern, and hence the particle distribution, was parabolic at points near to the mouth of the flow tube. Scans were therefore performed at a distance of 10 cm from the mouth of the tube, giving the distribution shown in figure 7. The central axial region appears to be parabolic in form, but there is an unexpected sharp concentration peak at about $r = 6{\cdot}75$ mm and a particle-depleted layer adjacent to the tube wall.

The question arose as to whether this arrangement was a property of the suspension flow or a measurement phenomenon. In order to resolve this a device consisting of a tangled piece of wire was placed in the mouth of the flow tube to produce turbulence. If the observed peaking were caused by a measurement phenomenon the mixing produced by the turbulence should make no difference

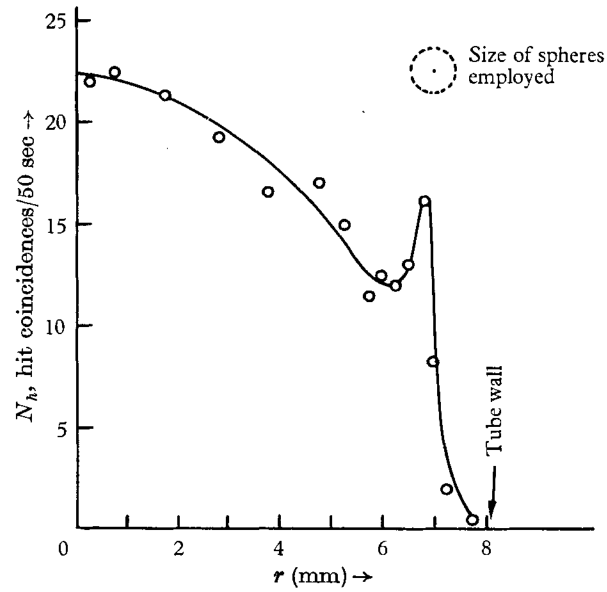

FIGURE 7. Graph of sphere count rate, in counts per 50 sec period at a distance $l = 10$ cm from the tube mouth at a tube Reynolds number of $Re = 830$ and a concentration of $C_0 = 4{\cdot}45$.

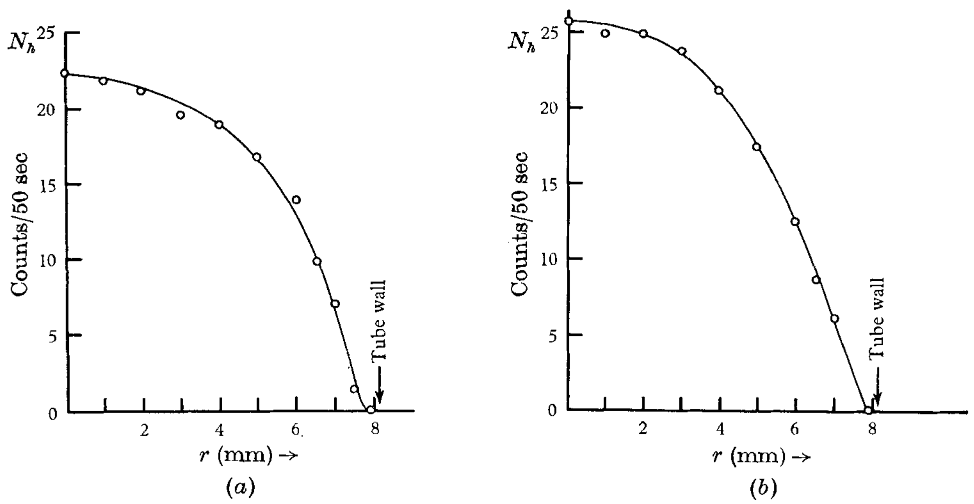

FIGURE 8. Similar to figure 7 except that turbulence was present at the tube mouth, and (a) $l = 10$ cm; (b) $l = 25$ cm.

to the measured particle distribution, while if it were indeed a property of the flow the peak should disappear. A scan performed under such conditions gave the distribution shown in figure 8 (a), which although not parabolic bears no trace of the peak. A scan at $l = 25$ cm shown in figure 8 (b) does have a parabolic nature.

These scans, actually performed in a tube of diameter 16·2 mm, indicate correct operation of the apparatus and that the observed peaking is a phenomenon of the flow.

In all cases considered in the present study, scans performed without turbulence at the mouth of the flow tube showed the presence of a narrow region of high concentration near to the tube wall, as in figure 9. (This was quite distinct from

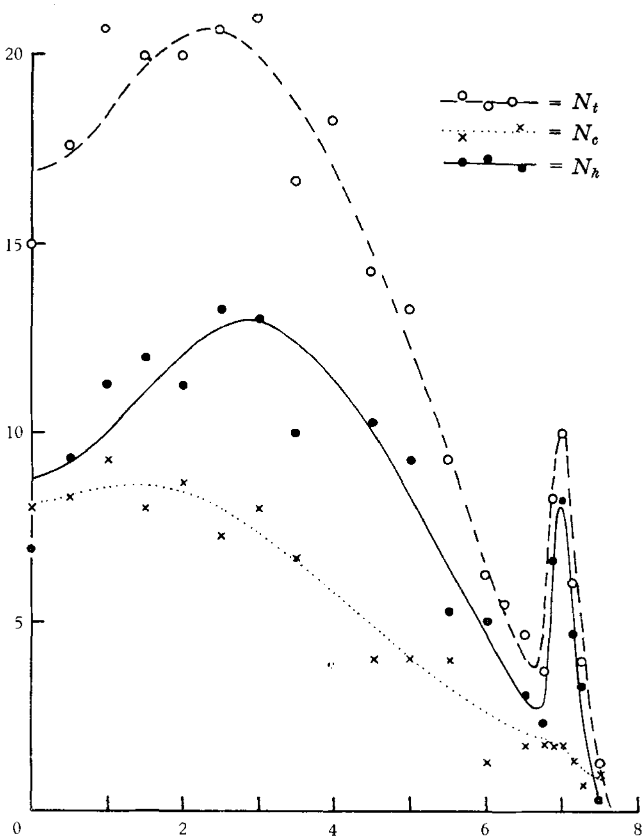

FIGURE 9. l = 78 cm, Re = 577, C_0 = 277 particle/cm^3 showing the development of both the 'tubular pinch' effect and the wall peak.

the 'tubular pinch effect' of the type described by Segré & Silberberg (1962).) This we have termed the 'wall peak', which other scans have shown can always be destroyed by the use of turbulence at the mouth of the tube. These results would suggest that this flow phenomenon has its origin in the region near to the mouth of the tube, rather than being created during the flow.

It would appear that this phenomenon has not previously been described, but because of its size and position it is likely to have a marked effect on viscosity measurements on the suspension, and as such warrants further investigation. Scans have therefore been performed to cover the particle distributions in this outer region of the tube under various conditions of flow, and at various distances from the mouth of the tube, such that the following information has been obtained regarding the nature and behaviour of the 'wall peak'.

The radial position of the 'wall peak' was found to be constant to within experimental error at all distances greater than 10 cm from the mouth of the flow tube under a wide variety of flow conditions. Figure 10 shows the distributions obtained from scans performed at the same distance from the mouth of the tube, and under flow conditions differing only in flow rate, while in figure 11 the flow conditions were similar but the scans were performed at different values of l. These results demonstrate the fact that at points away from the mouth of the tube the 'wall peak' lies at a constant radial position. It was not possible to investigate the peak position in the region very near to the tube mouth because

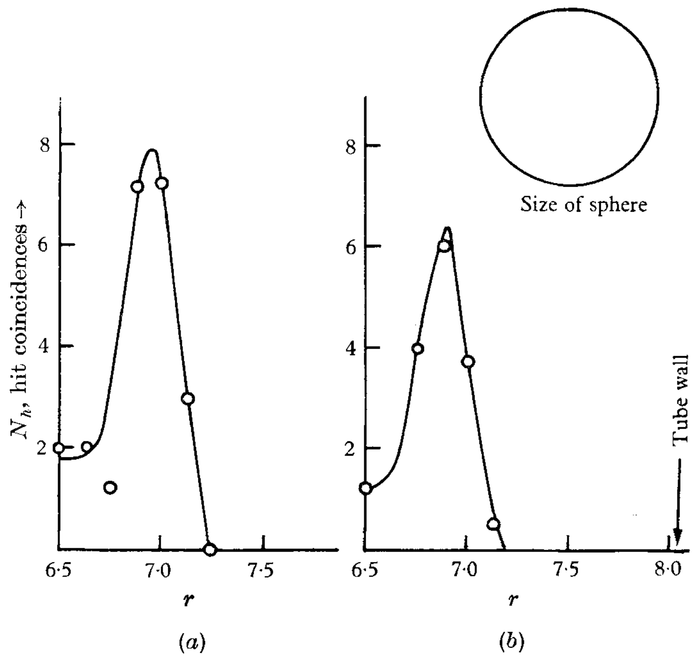

FIGURE 10. l = 58 cm, C_0 = 288 particle/cm^3, (a) Re = 710; (b) Re = 440 (wall peak only) showing the effect of rate of flow.

of the method of construction of the apparatus, which obscured this region. It can also be noted from these distributions that the region outside the 'wall peak' always consists of a particle-free layer of suspending fluid, indicating that the particles making up the concentration peak probably come from this region.

From figure 11 it is also possible to demonstrate the way in which the height of the 'wall peak' varies along the length of the flow tube. By plotting the height of the peak relative to the trough (i.e. $(N_h)_{\max}/(N_h)_{\min}$) against l, the distance from the mouth of the flow tube, the graph of figure 12 is obtained. Despite the experimental scatter, the points indicate a steady increase in the height of the peak on its passage down the tube. The solid curve, drawn in by eye, would appear to be a reasonable fit which apart from demonstrating the increase in height also indicates a peak of finite size at the mouth of the tube. Referring back to the beginning of this section it was shown that the use of a turbulence producer at the mouth of the tube effectively removed the 'wall peak' so that both results indicate that the

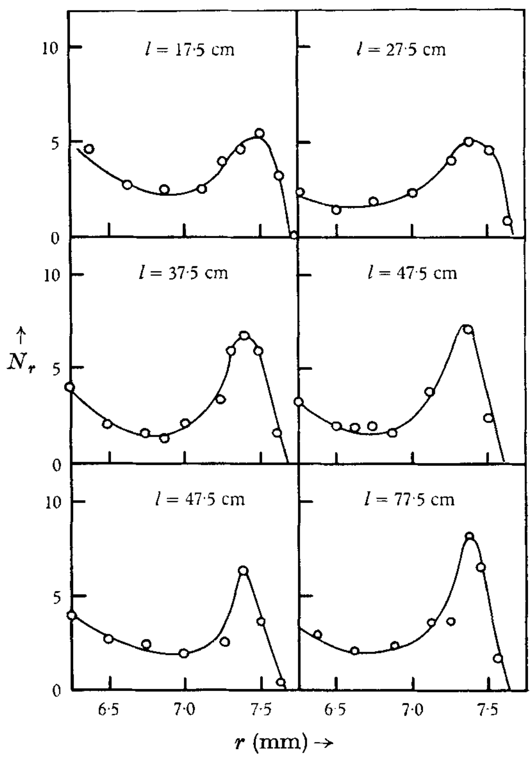

FIGURE 11. Development of the wall peak along the tube. $C_0 = 2{\cdot}90$ particle/cm^3, $Re = 540$.

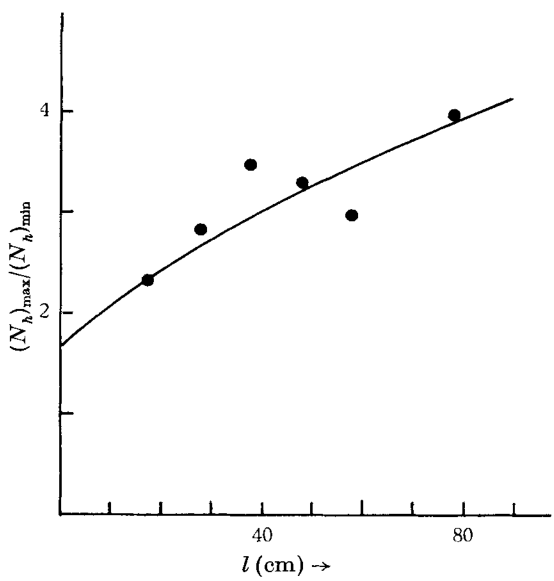

FIGURE 12. Variation of height of the wall peak along the length of the flow tube at $C_0 = 2{\cdot}90$ particle/cm^3, $Re = 540$.

origin of the peak lies in the region of the tube mouth. From these results it would appear that, whilst the initial creation of the peak takes place somewhere in the region of the mouth of the tube, it continues to develop during the passage down the flow tube.

A mechanism suitable for the initial creation of the peak has been described by Maude & Whitmore (1956), who point out that in order to enter the flow tube certain particles lying on streamlines adjacent to the lip of the flow tube must be mechanically displaced inwards. Such a process of mechanical interaction predicts an initial inwards displacement of one particle radius which is somewhat smaller than that shown in the present case, although it has already been pointed out that it has not been possible to observe the position of the peak immediately after the entrance. It is proposed however that the 'wall peak', which has been observed to occur in tube flow under a wide range of conditions, is created by the process of interaction with the tube mouth, resulting in an inward displacement of particles and also giving rise to a particle-free layer of suspending fluid at the tube wall. The extra displacement may be due to a more complex interaction than that proposed by Maude & Whitmore, or to further movement occurring immediately below the tube mouth. Figures 11 and 12 indicated a gradual increase in the height of the peak on passage down the tube, which is accompanied by a narrowing of the peak. Study of the peaks in figure 11 indicates that this increase in height is caused by particles migrating into this region from faster-moving streamlines, which means that the peak must have some internal structure capable of attracting in other particles, although these results do not in themselves yield any information regarding the actual peak structure or the nature of the migrations.

A concentration peak of this size would be expected to have a large effect on the viscosity measurements performed in a capillary tube viscometer, but the present readings have only been performed at low particle concentrations where changes in relative viscosity could not be determined with any accuracy. It is therefore necessary that the effects of particle–particle interaction brought about by an increase in concentration be investigated, before this can be in any way related to the so-called 'anomalous viscous properties' of suspensions. A series of scans was therefore performed in which all parameters were kept constant with the exception of concentration, which was varied in 13 steps between 2·88 and 39·2 particles/cm^3. The results of six of these, chosen at convenient intervals of concentration, are shown in figure 13, and clearly demonstrate the way in which the increased particle concentration destroyed the wall peak. In order to demonstrate this more clearly the graph has been drawn in which the height of the peak relative to the trough is plotted against concentration (figure 14). Where the peak or trough are ill defined the values are taken as those of $r = 7{\cdot}25$ and $6{\cdot}75$ mm respectively. Even at the lowest concentrations employed it is seen that increase of concentration has had an effect on the size of the peak, although it must be remembered that even here, because of its large size, the concentration within the peak will be quite high compared to the overall concentration. At overall concentrations up to about 11 particles per cm^3 the rate of reduction of peak size with increased concentration is large but above this value the rate of decline is reduced. Such an occurrence would be consistent with the presence of two sets of forces, one due to particle interaction tending to destroy the peak, which it does efficiently up to about 11 particles per cm^3, and another as yet unspecified interaction which tends to hold the peak together particularly at high concentrations.

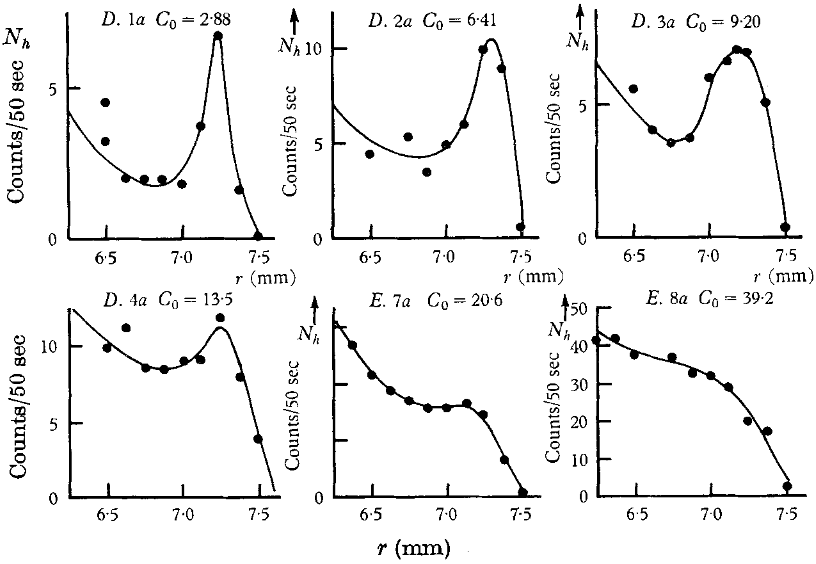

FIGURE 13. The effect on the wall peak of increase of concentration. L = 52 cm, Re = 540.

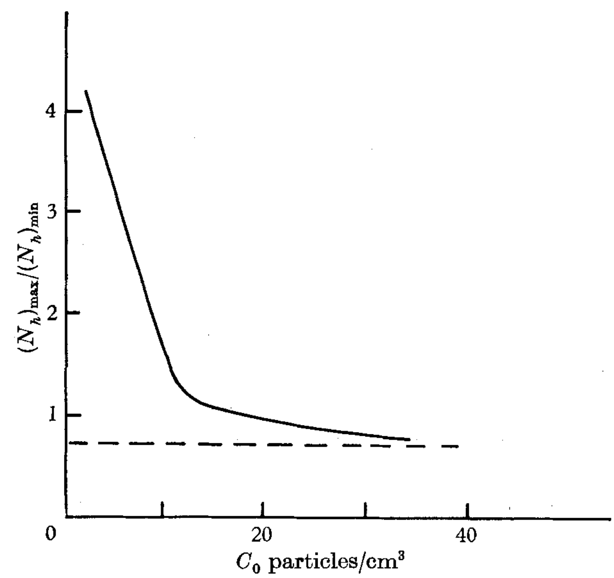

FIGURE 14. Variation of height of the wall peak with concentration.

The latter part of the curve in figure 14 appears to tend asymptotically to the horizontal line drawn at $(N_h)_{max}/(N_h)_{min} = 0{\cdot}73$, which is the value at which the pattern becomes parabolic. This figure is arrived at by assuming a parabolic velocity profile and that the particles follow the local fluid velocity at their centres, i.e.

$$\frac{(N_h)_{\max}}{(N_h)_{\min}} = \frac{(v_z(r))_{\max}}{(v_z(r))_{\min}} = \frac{(R^2-(r)^{22}_{\max})}{(R^2-(r)^2_{\min})} = \frac{(8\cdot45)^2-(7\cdot25)^2}{(8\cdot45)^2-(7\cdot75)^2} = 0\cdot73.$$

However, when figure 13 is examined carefully, and it is realized that the tube wall lies at a position of $r = 8\cdot45$ mm, it is seen that although the pattern in the region of the 'wall peak' tends to return to a parabolic nature this is not so for the tube as a whole, since there is still a particle-free layer of suspending fluid adjacent to the tube wall.

These results provide a further method for checking the proposition that the process causing the wall peak is that of mechanical interaction at the mouth of the

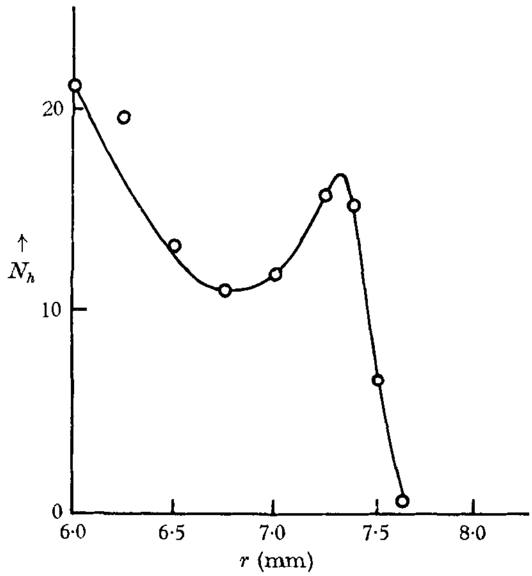

FIGURE 15. l = 12 cm, C_0 = 10·35 particle/cm³, Re = 570.

flow tube. It is seen from figure 14 that at an overall concentration of 10 particles/cm³ the peak will have virtually disappeared at a distance of 52 cm from the mouth of the tube, but if the proposed mechanism for the creation of this peak is correct, the setting up at the mouth of the tube should be unhindered by any interaction effects. A scan was therefore performed at a distance of 12 cm from the mouth of the tube at a concentration of 10·55 particles/cm³, the resulting distribution being shown in figure 15. This clearly demonstrates the existence of a well-formed wall peak supporting the proposition of interaction with the tube mouth for its initial creation.

In view of these results it is therefore to be expected that the effects of disruptive particle interaction, caused by an increase in particle concentration, will cause a steady reduction in the height of the wall peak along the length of the flow tube. Scans were therefore performed at 20 cm separations along the length of the flow tube at a concentration of 10·55, the results being shown in figure 16.

As expected the first three distributions obtained in this set indicate a steady decrease in the height of the peak, but the final two in the lower part of the tube show a further unexpected increase. Such a situation poses the problem that in

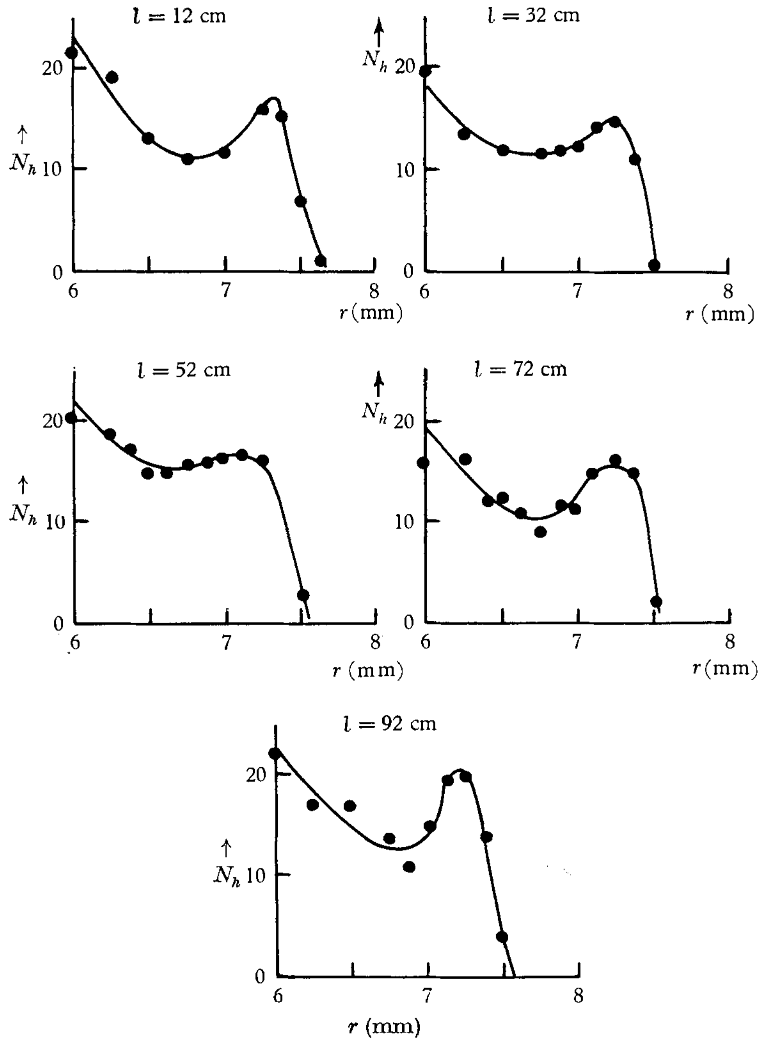

FIGURE 16. Development of the wall peak along the length of the flow tube at C_0 = 10·55 particles/cm^3, *Re* = 540.

order for this to occur there must be present some other set of forces in the latter part of the flow which tends to increase the size of the peak, and which must be strong enough to offset the disruptive forces of particle interaction. It is possible that these forces are the ones responsible for the change in slope of the graph in figure 14 and it appears that an explanation must be sought by considering the internal structure of the wall peak.

4. Particle behaviour within the flow

During the execution of this study the 'necklace-like formations of particles' reported by Segré & Silberberg (1962), and studied by Segré (1963), were a common feature of the flow. These chains of particles were observed to form in the lower parts of the flow tube and consisted of two or more particles in line separated by an apparently uniform distance of the order of one particle diameter.

Visual observation of these formations showed clear cases of particle capture where a particle approaching a chain from a faster-moving streamline was pulled on to the rear end of the chain, thereby increasing its length. When such a process occurred the chains were observed to 'rock' with longitudinal oscillations until equilibrium was reached. Capture was also observed to occur of one complete chain by another in a similar manner. Observation over long periods gave the impression of the amazing stability of the chains, which would maintain their form even during quite violent rocking, although occasionally one or more particles were observed to leave the downstream end of a long chain.

Despite the similarities between the present case and that of Segré & Silberberg (1962) and Segré (1963), the slow-moving nature of the chains and their apparent nearness to the tube wall made it difficult to believe that they could be in a position associated with the 'tubular pinch effect'. In order to locate the radial position of these chains it was therefore decided to measure their velocities at a known flow rate and relate this to position by assuming a parabolic velocity profile.

The flow system shown in figure 1 was again used but in this case the flow tube together with reservoirs C_1 and C_2 were immersed in a long rectangular Perspex tank filled with water to enable visual observation over the entire length of the flow tube. Using a Pathé Webo 16 mm ciné camera operating at 80 frames/sec a film was made at a point of the tube where chains existed. In the field of view of the camera there was also a stop-watch and a metre rule parallel to the flow tube. Projection of the film at 16 frames/sec resulted in an apparent slow motion of the chains which were clearly visible in the slower-moving parts of the flow. None were observed in the central region of the tube although this does not exclude the possibility of their existence here, since even under the present circumstances the particles were still moving fast enough to make it difficult for the eye to follow over the short length of tube within the field of view. Measurement from individual film frames made it possible to find the distances travelled by the chains in known periods, and in this way the velocities were calculated for 89 cases. The distribution of these is shown in figure 17. The mean velocity was calculated as 8·90 cm/sec, which is assumed to be the value for the peak of the distribution. In this case the flow rate employed was $Q = 37{\cdot}5\,\text{cm}^3/\text{sec}$, which, when using the equation for parabolic flow, shows that 8·9 cm/sec corresponds to a radial position of $r = 7{\cdot}24$ mm, which is to within 0·1 mm of the position of the wall peak as measured by the optical scanner. No other chains were detected by this means and the results are therefore consistent with the existence of a concentration peak consisting of chains in the region of the wall peak.

As a control the velocities of single particles were also measured in the same way as for the chains. The distribution is also shown in figure 17. The cut-off of particles observed at high speeds is an indication of the limit of this method of observation. The comparative ease of measurement of the slower-moving particles leads to the selection of a biased sample, but the method does provide the following confirmatory evidence. The fact that in the case of individual particles the

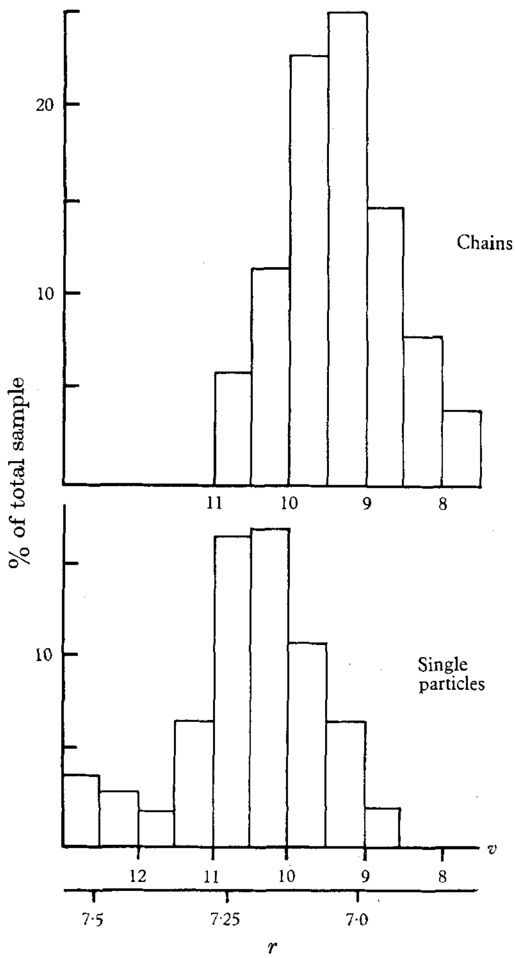

FIGURE 17. Distribution of chains and single particles near to the tube wall.

peak lies at a smaller radial position than for the chains indicates that the distributions are separate and that the fall in the number of chains is quite genuine and not due to any lack in ability to count them. In other words, it is evident that there does in fact exist a peak consisting of chains in a position indicated by the optical scanner. It is also seen that no particle, either singly or in chains, was observed with a velocity less than 7·0 cm/sec, confirming the results obtained from the optical scanner that there exists a particle-free layer of suspending fluid adjacent to the tube wall.

In view of the localization of the particle chains within the wall peak it is now possible to suggest a physical model for this region, and the way it might develop under various conditions.

Consider first a suspension at low concentration, say of the order of 3 particles/cm^3, flowing through the tube. As the particles enter the flow tube those near to the lip will be displaced by a process of mechanical interaction in the manner already described. If this process is exactly as predicted by Maude & Whitmore (1956) the peak would lie nearer to the wall and it must therefore continue to move rapidly inwards to the observed final position, either as a result of its own momentum acquired in the mechanical interaction or under the influence of hydrodynamic forces. The interaction with the tube mouth may however be more complex than a simple mechanical contact, and the peak may be formed a short distance from the wall. A ciné film of this region of the flow tube near to the mouth indicates the existence of no chains, so that here the peak of concentration has no internal structure. However, the creation of such a region of high concentration is conducive to the formation of chains (Segré 1963) which will start to appear as the suspension flows down the tube. Once a chain has managed to form it is a stable arrangement and capable of attracting in particles from adjacent streamlines. Such a process results in an increase in the length of the chains and a consequent increase in the height of the peak. Such a process might be expected to continue either until all the particles have been attracted in from adjacent streamlines, or until the disruptive forces of particle interaction are strong enough to counteract the effect. Under certain circumstances the situation can be visualized where the tubular pinch effect and the wall peak become completely dissociated, being separated by a layer of clear suspending fluid.

The existence of chains can also be used to explain the behaviour of the wall peak at higher concentrations in the following manner. At concentrations of the order of 10·55 particles/cm^3, which was operative when the graphs of figure 16 were obtained, the wall peak will still be formed at the mouth, but on flowing down the tube will start to be dispersed by the disruptive effects of particle interaction. However at the same time hydrodynamic forces will be acting on the particles in the peak which will be tending to form chains. Even at low concentrations chains are not observed to form before about $l = 60$ cm and it is in this region at the higher concentration that they will also form. Once the chains are formed the peak will become more stable and start to increase in size again due to increase in the length of the chains. Visual observation of the flow tube supports this proposition in that even at the highest concentrations employed in the present study the chains were observed in the latter part of the flow.

5. Conclusions

The results of the present experimental study have enabled at least a qualitative picture to be built up of the existence and behaviour of the phenomenon which we term the 'wall peak'. The results available suggest that this arises as a result of interaction of particles with the mouth of the tube, and that its behaviour is thereafter governed by the creation and development of the 'necklace-like formation of particles' which are observed within the flow. At the present moment we are not in a position to state what factors affect the final radial position of the peak, whether it is determined purely by the particle radius or whether the tube dimension is also important. It is hoped that in the near future work being carried out in this Department may solve this question.

Although the wall peak is very prominent at low particle concentrations, at concentrations where measurements of relative viscosity are practically feasible it is not likely to be present, and cannot therefore be regarded as an important feature in explaining the so-called 'anomalous viscous properties' of suspensions. Whether the particle-free layer at the tube wall will also disappear at high concentrations cannot be resolved on the basis of the present study.

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